Faculty of Science, Technology, Engineering and Mathematics
MST210 Mathematical methods, models and modelling

MST210

TMA 03 2019J

Covers Units 4, 5, 6 and 7 **Cut-off date 11 December 2019**

The module website gives details of how to submit assignments for this module. If you have a disability that makes it difficult for you to attempt any of the questions, then please contact your Student Support Team or your tutor for advice.

The marks allocated to each part of each question are indicated in brackets in the margin.

You may use Maxima to check answers, but it should not be used otherwise to answer TMA questions except where explicitly indicated. TMA answers must show full working to attract full marks.

In order to encourage you to present your solutions to the TMA questions in a good mathematical style, your tutor will comment on how you:

- use correct mathematical notation
- define any symbols that you introduce in formulating and solving a problem
- give references for standard formulas and derivations
- include comments and explanations within your mathematics
- explicitly state results and conclusions, giving answers to an appropriate degree of accuracy and interpreting answers in the context of the question
- draw diagrams and graphs.

These features are seen as being essential to complementing your mathematical skills. Your tutor will make comments on how well you achieve these objectives and give you guidance on how to satisfy them.

Five of the marks for this TMA will be allocated to the way you write your solutions. It is expected that most students will receive the majority of these presentation marks; such marks are included in TMAs to encourage, and emphasise the need for, thinking about how you present your mathematics.

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Question 1 (Unit 4) – 9 marks

Consider the following system of equations:

$$\begin{aligned} 2x - y + 0 &= 4, \\ -4x + 3y + 4z &= -2, \\ 2x - 4y - 3z &= -5. \end{aligned}$$

Use the Gaussian elimination method to reduce the system to upper triangular form. Carry out your calculations by hand using exact arithmetic, using the methods of Subsections 1.2 and 1.3 of Unit 4. Clearly label the operations that you use. If the set of equations has no solution, then clearly state this; if it has a unique solution, then find it; and if it has an infinite number of solutions, then find the general solution. [*9*]

Question 2 (Unit 4) – 9 marks

Consider the following system of equations:

$$\begin{aligned} 2x + y - z &= -1, \\ 4x - 2y + 2z &= 2, \\ -2x - 5y + 5z &= 5. \end{aligned}$$

Use the Gaussian elimination method to reduce the system to upper triangular form. Carry out your calculations by hand using exact arithmetic, using the methods of Subsections 1.2 and 1.3 of Unit 4. Clearly label the operations that you use. If the set of equations has no solution, then clearly state this; if it has a unique solution, then find it; and if it has an infinite number of solutions, then find the general solution. [*9*]

Question 3 (Unit 4) – 5 marks

Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & -2 \\ 3 & 1 & 2 \\ 2 & 1 & -4 \end{pmatrix}.$$

(a) Calculate the determinant of $\mathbf{A}$. [*3*]

(b) State whether $\mathbf{A}$ has an inverse. Briefly justify your answer. [*2*]

Question 4 (Unit 5) – 26 marks

Consider the matrix

$$\mathbf{A} = \begin{pmatrix} -3 & 0 & 0 \\ -6 & 4 & 1 \\ -8 & 6 & -1 \end{pmatrix}.$$

(a) Show that the eigenvalues of $\mathbf{A}$ are -3, -2 and 5. For each eigenvalue, find a corresponding eigenvector. [*13*]

(b) Find the eigenvalues of $-3\mathbf{A}$, $\left(\mathbf{A}^{-1}\right)^2$ and $2\mathbf{A}^{-1} + \mathbf{I}$, justifying your answers. [*6*]

(c) Using Maxima, or otherwise, carry out 50 iterations of the direct iteration procedure described in Procedure 4 on page 107 of Unit 5 on the matrix $\mathbf{A}$ in order to estimate the eigenvalue of largest magnitude. Your answer should include the computer input, together with computer output (including a plot of progress of the iteration, as in the computer session). You should annotate your answer to indicate the main features of the input and output.

(*Hint*: Modify the code given in the computer session for Unit 5.) [*2*]

(d) Using Maxima, or otherwise, carry out 50 iterations of the modified inverse iteration procedure described in Procedure 6 on page 111 of Unit 5 on the matrix $\mathbf{A}$ with $p = \frac{3}{2}$ in order to try to find the eigenvalue closest to $\frac{3}{2}$. Your answer should include the computer input, together with computer output (including a plot of progress of the iteration, as in the computer session). You should annotate your answer to indicate the main features of the input and output.

Explain why the method fails to converge, and suggest a change in the parameter p that will lead to convergence to the eigenvalue of smallest magnitude.

(*Hint*: Modify the code given in the computer session for Unit 5.) [*5*]

Question 5 (Unit 6) – 17 marks

Find the solution of the simultaneous linear differential equations

$$\frac{dx}{dt} = 5x - 3y + 11e^{-2t},$$

$$\frac{dy}{dt} = 2x - 2y + 4e^{-2t}$$

that satisfies the initial conditions $x(0) = -4$, $y(0) = 0$. [*17*]

Question 6 (Unit 6) – 6 marks

Consider the system of differential equations

$$\ddot{x} = -7x + 9y,$$
$$\ddot{y} = 9x + 17y.$$

Given that the coefficient matrix $\begin{pmatrix} -7 & 9 \\ 9 & 17 \end{pmatrix}$ has eigenvectors $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$, find the general solution of this system. [*6*]

Question 7 (Unit 7) – 23 marks

Consider the function

$$f(x, y) = 8x^2 + xy^2 + 6xy - 7x + 8.$$

(a) Find the stationary points of f. [*9*]

(b) Classify each of the stationary points. [*7*]

(c) Determine the second-order Taylor polynomial for f near $(2, 1)$. (There is no need to expand any brackets that may appear in your answer.) [*4*]

(d) The value of f when $x = 2$ and $y = 1$ is 40. Suppose, however, that we know only that the value of x lies between 1.98 and 2.02, while the value of y lies between 0.99 and 1.01. Use first-order partial derivatives to investigate the accuracy of 40 as an estimate of $f(x, y)$ for values of x and y in these ranges. [*3*]